Vortex Production and Longevity in Protoplanetary Disks

The process of planetary formation is not well understood. Observations show that young stars are surrounded by disks of gas and dust. These disks, called protoplanetary or accretion disks, must be the source of material for both terrestrial planets and gas giants. The question is how such diffuse, tenuous gas and dust grains could form planets like those we see today.

Current theories such as core accretion and gravitational instability do not consider the influence of turbulent dynamics in planetary formation. Turbulence and long-lived vortices can significantly alter the density of solids that is critical for both accretion and instability processes. Simple analytic and numerical models show that gas vortices in protoplanetary disks are extremely efficient at concentrating particles. Inside of the vortices the surface density increases by several orders of magnitude, speeding the process of planetary formation.

We have derived a reduced equation set for vorticity and entropy from the full conservation equations to study vortex formation and longevity in cool (nonmagnetic) disks [1]. The model is two-dimensional and anelastic, so that the background density varies radially but not in time.

The essential dynamical terms in the equation set are baroclinicity and radiative cooling. Baroclinicity is the only source of vorticity in two-dimensional turbulence, and is required to study vortex formation. Baroclinic effects on the disk are produced by azimuthal temperature gradients interacting with the background radial temperature and density profiles. Thus a prognostic energy equation and thermal effects like radiative cooling are also required. Several other studies do not include these terms, and therefore only study decaying turbulence from some initial vorticity field.

A linear stability analysis of the reduced equations shows that instabilities occur when the background temperature and density gradients are sufficiently large. Including radiative cooling in this analysis causes stronger instabilities.

I created a numerical model which is fast and efficient due to several wise choices in the derivation of the equations and design of the numerical model [1]. These include: (1) a reduced equation set with only two prognostic variables; (2) the anelastic regime, which filters out pressure waves; (3) vertical integration of the equation set, which results in a 2D model; (4) the use of a pseudospectral numerical method, with its exponentially convergent spatial representation; (5) reduction of the Laplacian operator from a triangular matrix to a banded diagonal of width six, and (6) a semi-implicit Runge-Kutta scheme that allows large timesteps. The model uses the Tau method and the influence matrix technique to enforce zero thermal flux and stress-free boundary conditions.

I am interested in exploring the necessary conditions for vortex formation in protoplanetary disks. Hundreds of simulations were run at moderate resolution of 256^2 in order to explore an unprecedented swath of parameter space. Higher resolutions of 512^2 were used for particular cases of interest.

Numerical simulations show that long-lived vortices can be formed by initial random temperature perturbations through the mechanism of the baroclinic instability. Vortex merger is the physical mechanism that transfers energy to larger scales in two-dimensional turbulence, a process known as the inverse energy cascade. Vortex production in protoplanetary disks must compete with the strong inhibiting effects of Keplerian shear; cylones shear out, while anticyclones remain coherent and merge with other anticyclones (Figure 1). (Cyclonic vortices rotate the same direction as the background Keplerian rotation, while anticyclones are opposite.) In order for vortices to form, baroclinic vorticity production must be sufficiently strong to overcome shearing and dissipation. The baroclinic production term includes factors of radial temperature and density gradients. To form vortices, the initial temperature perturbation must be at least 10% of the maximum background temperature, and the background temperature or surface density profile must be sufficiently steep (Figure 2). These general results were predicted by the linear stability analysis of the reduced equation set.

The dynamics of the disk is highly sensitive to radiative cooling. Simulations with slow cooling are characterized by a few orbital periods of baroclinic vortex production followed by vortex merger and decay. The thermal field quickly forms a radial stratification, and a lack of azimuthal temperature variation stops the baroclinic production. In the case of fast cooling the temperature and vorticity fields are closely coupled, leading to large, long-lived vortices [2].

The transport of angular momentum is of critical interest in the study of protoplanetary disks. The traditional view of disk evolution is that angular momentum is transported outward as mass is transported inward. This theory of outward angular momentum transport is based on azimuthally uniform dynamics in a viscous disk. Turbulence and coherent structures may have radically different effects. We found that angular momentum transport could be either positive or negative, and that an individual anticyclone

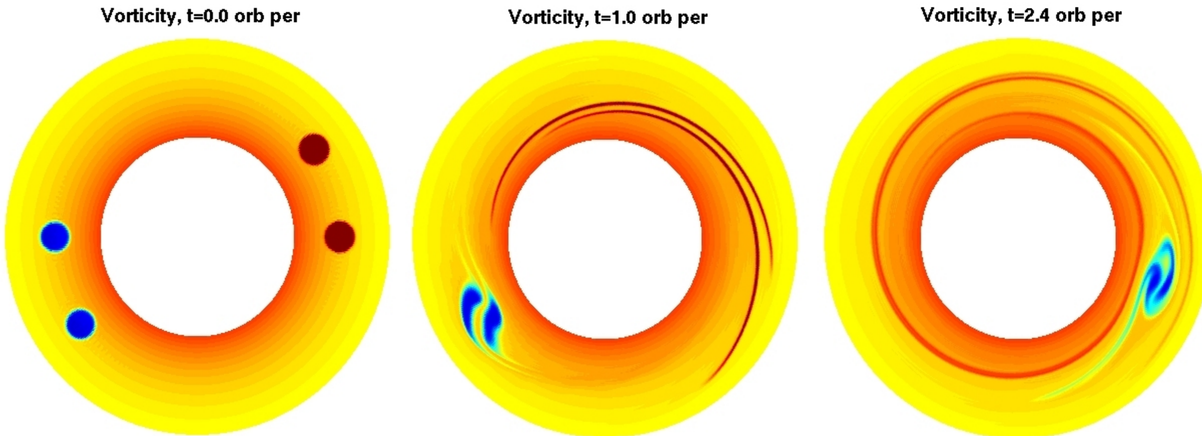

Figure 1: The reaction of vortices to background shear. Cyclones (positive, red) shear out while anticyclones (negative, blue) remain coherent and merge. The differential background rotation is shown as the background gradient from red to yellow.

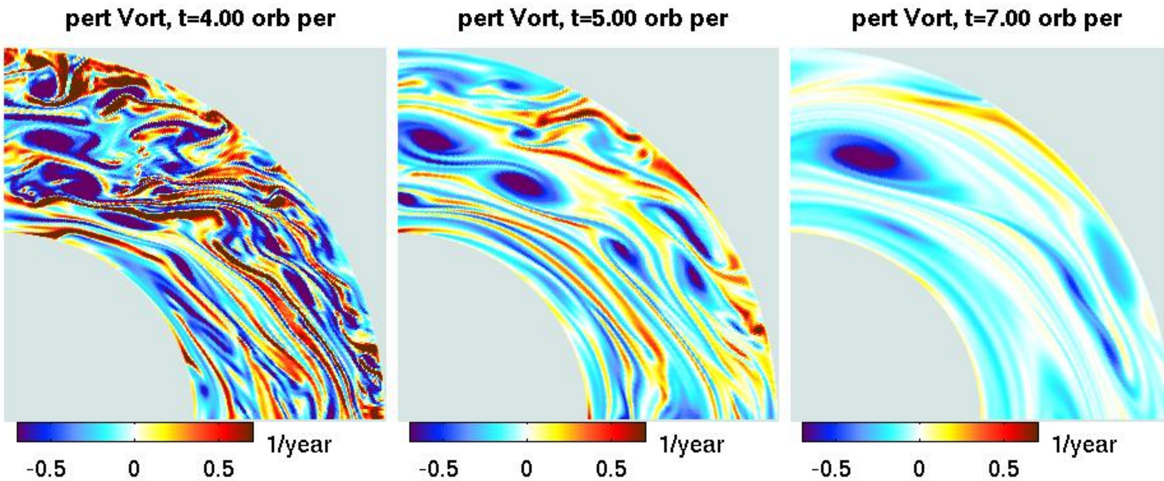

Figure 2: Perturbation vorticity in a protoplanetary disk simulation where the background radial temperature and density gradients were sufficiently steep to initiate the formation of vortices. Initial conditions were zero perterbation voricity and a random temperature perturbation.

is usually associated with negative angular momentum transport. This result is supported by a simple vortex model which shows that the sign of the angular momentum transport depends only on the orientation of the vortex.

In summary, I have shown that baroclinicity can be a significant source of vorticity in protoplanetary disks if temperature perturbations are strong and radial temperature and density gradients are large. Additionally, the characteristics of vortex formation are sensitive to the rate of radiative cooling, and radiative cooling can enhance the effects of baroclinic vorticity production to maintain vortices for over 120 orbital periods.

References

[1] Mark R. Petersen, Keith Julien, and Glen R. Stewart. Baroclinic vorticity production in protoplanetary disks; Part I: Vortex formation. *Astrophys. J.*, 658:1236–1251, 2007.

[2] Mark R. Petersen, Glen R. Stewart, and Keith Julien. Baroclinic vorticity production in protoplanetary disks; Part II: Vortex growth and longevity. *Astrophys. J.*, 658:1252–1263, 2007.

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